

# THE ENERGY OF LONG DURATION GRBS

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## ABSTRACT

The energy release in gamma-ray bursts is one of the most useful clues on the nature of their “inner engines”. We show that, within the framework of the relativistic external shocks afterglow model, the narrowness of the observed X-ray luminosity of GRB afterglows, implies that the energy of the GRB jets after the early afterglow phase spans less than one order of magnitude. This result is not affected by uncertainties in the electrons energy, magnetic field strength, and external medium density. We argue that the afterglow kinetic energy is within a factor of two of the initial energy in the relativistic ejecta, therefore the energy output of the central engine of long duration GRB has an universal value.

Subject headings: gamma-rays: bursts - ISM: jets and outflows - radiation mechanisms: non-thermal - shock waves

In the last three years we have learned a great deal about long duration gamma-ray bursts (GRBs). The Italian-Dutch satellite BeppoSAX provided angular position of several dozen long bursts to within about 3 arc-minutes which enabled follow up observations in the x-ray (see Piro, 2000), optical, milli-meter and radio frequencies which has provided a wealth of information on these explosions. These observations are described well by the relativistic fireball model (see e.g.

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Piran, 1999). According to this model the energy from the central source is deposited in material that moves with speed very close to the speed of light. The kinetic energy of this material is converted to the observed electromagnetic radiation as a result of collisions between fast moving material that catches up with slower moving ejecta, and the shock heated circum-burst medium. The nature of the “inner engines” that expels relativistic material, which is responsible for the the GRBs, is not determined yet. The energy of the relativistic matter ejected by the “inner engine”,  $E_{rel}$ , is one of the most important clues on its nature. Our goal is to find a reliable estimate of  $E_{rel}$ .

Given an observed  $\gamma$ -ray fluence and the redshift to a burst one can easily estimate the energy emitted in  $\gamma$ -rays,  $E_{\gamma,iso}$  assuming that the emission is isotropic.  $E_{\gamma,iso}$  can also be estimated from the BATSE catalogue by fitting the flux distribution to theoretical models (Cohen & Piran, 1995; Schmidt, 2001). As afterglow observations proceeded, alarmingly large values (Kulkarni et al. 1999) ( $3.4 \times 10^{54}$  ergs for GRB990123) were measured for  $E_{\gamma,iso}$ . However, it turned out (Rhoads, 1999; Sari Piran & Halpern, 1999) that GRBs are likely beamed and  $E_{\gamma,iso}$  would not then be a good estimate for the total energy emitted in  $\gamma$ -rays. We define instead:  $E_{\gamma} = (\theta^2/2)E_{\gamma,iso}$ . Here  $\theta$  is the effective angle of  $\gamma$ -ray emission, which can be estimated from  $t_b$ , the time of the break that appears latter in the afterglow light curve (Rhoads, 1999; Sari Piran & Halpern, 1999):  $\theta = 0.12(n/E_{51})^{1/8}t_{b,days}^{3/8}$ , where  $E_{51}$  is the isotropic-equivalent energy kinetic energy during the adiabatic fireball phase, discussed below, in units of  $10^{51}$  ergs, and  $t_{b,days}$  is the break time in days. Recently Frail et al. (2001, hereafter F1) estimated  $E_{\gamma}$  for 18 bursts, finding typical values around  $10^{51}$  ergs. While  $E_{\gamma}$  is closer to  $E_{rel}$  it is still not a good estimate. First, we have to take an unknown conversion efficiency of energy to  $\gamma$ -rays into consideration:  $E_{rel} = \epsilon^{-1}E_{\gamma} = \epsilon^{-1}(\theta^2/2)E_{\gamma,iso}$ . Second, the large Lorentz factor during the  $\gamma$ -ray emission phase, makes the observed  $E_{\gamma}$  rather sensitive to angular inhomogeneities of the relativistic ejecta (Kumar & Piran, 2000).

We consider here another quantity:  $E_{K,ad}$ , the kinetic energy of the ejecta during the adiabatic afterglow phase<sup>¶</sup>. Clearly:  $E_{rel} \geq \overline{E_{\gamma}} + E_{K,ad} = \epsilon E_{rel} + E_{K,ad}$ , where  $\overline{E_{\gamma}}$  is the angular average of  $E_{\gamma}$ . The inequality arises from possible energy losses during the early afterglow radiative phase. However observations of long time tails of GRBs suggest that, unless this energy is radiated at extremely high energy channel, this losses are not large (Burenin et al, 1999, Giblin et al, 1999; Tkachenko et al., 2000) . Therefore, with  $\epsilon \approx 10\%$  (Kobayashi, Piran & Sari, 1997) ( $\epsilon$  cannot be too close to unity otherwise there won’t be afterglow) we expect that  $E_{K,ad} \approx E_{rel}$  to within better than a factor of 2. Hereafter we drop the subscript *kin* denoting  $E = E_{K,ad}$ .

The purpose of this paper is to determine the spread of  $E$  using the x-ray afterglow flux. The advantage of the method presented here is that it is independent of the uncertain density of the ISM, and in fact all other parameters, except for the observed width of the distribution of the jet opening angle.

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<sup>¶</sup>The external relativistic shock becomes adiabatic about 1/2 an hour after the explosion, and furthermore the loss of energy during the earlier radiative phase is typically not large.

One way of determining how  $E$  is distributed is to figure out the energy for individual bursts by modeling their afterglow emission over a wide range of frequency and time. This procedure, carried out for 8 well studied bursts (Panaitescu & Kumar 2001, hereafter PK01), gives the mean energy to be  $\sim 3 \times 10^{50}$  ergs and the standard deviation of the log of energy distribution to be about 0.3. However, the detailed modeling of individual GRB afterglow emission is cumbersome and time consuming, and hard to carry out for a large sample of bursts especially considering that we need data in radio, optical and x-ray bands with good time coverage for this kind of an analysis to be useful.

Moreover, this procedure is not necessary if we simply want to know the width of energy distribution. For this we can use the x-ray afterglow flux at a fixed time after the explosion. The width of the distribution of this flux, an easily measurable quantity, yields the width of the distribution for the energy release in the explosion. This method is described below. The observation of the x-ray flux should be carried out at a sufficiently late time such that the angular variations and fluctuations across the surface of the ejecta have been smoothed out. This occurs several hours after the explosion when the bulk Lorentz factor of the ejecta has decreased to about 10 at which time we see a good fraction of the relativistic ejecta. Conveniently, this is also when the observed x-ray in the 2-10 keV band is above the cooling frequency in which case the observed flux is independent of the density of the medium in the vicinity of the explosion (Kumar 2000). Furthermore, it is best to carry out observations while  $\Gamma > \theta^{-1}$ , that is before the jet begins its sideways expansion, which makes the interpretation of the observed flux much simpler. In five cases of GRB afterglow light curves where we see the effect of the finite opening angle of explosion as steepening of the light curve, we find the effect manifests itself at least one day after the explosion. Thus, the above two requirements suggest it is best to consider the x-ray afterglow flux between several hours and a day after the explosion.

The x-ray afterglow fluxes from GRBs have a power law dependence on  $\nu$  and on the observed time  $t$  (Piro, 2000):  $f_\nu(t) \propto \nu^{-\beta} t^{-\alpha}$  with  $\alpha \sim 1.4$  and  $\beta \sim 0.9$ . The observed x-ray flux per unit frequency,  $f_x$ , is related, therefore, to,  $L_x$ , the isotropic luminosity of the source at redshift,  $z$  by:

$$L_x(t) = \frac{4\pi d_L^2}{(1+z)} f_x(t) (1+z)^{\beta-\alpha} \equiv f_x(t) Z(z) , \quad (1)$$

where  $Z(z)$  is a weakly varying function of  $z$ . For bursts with  $0.5 < z < 4$  and with  $\beta - \alpha \approx -0.5$  we find  $\sigma_Z \approx 0.31$  (for a cosmology with  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$ ). Here and thereafter we denote by  $\sigma_X$  the standard deviation of the  $\log(X)$ , unless noted otherwise.

Assuming that the x-ray luminosity does not evolve with redshift we can relate the dispersion of  $\log(L_x)$  at a fixed observer time after the explosion,  $\sigma_{L_x}$ , with the observed dispersion  $\sigma_{f_x}$ :  $\sigma_{L_x}^2 = \sigma_{f_x}^2 + \sigma_Z^2 \approx \sigma_{f_x}^2$ . Using 21 BeppoSAX bursts (Piro, 2000) we find  $\sigma_{f_x} \approx 0.43 \pm 0.1$  (see the caption for Figs. 1 and 2 for the details of the observations and the analysis), and therefore,  $\sigma_{L_x} \approx 0.43$  to within 25%. This result is supported by 10 x-ray light-curves of GRBs with known red-shifts, and  $\alpha$  and  $\beta$ .

The x-ray flux, in an energy band above the cooling frequency at a fixed time after the burst, depends on the energy per unit solid angle in the explosion (provided that  $\Gamma > \theta^{-1}$  at the time of observation), and on the fractional energy taken up by electrons,  $\epsilon_e$ . The flux does not depend on the density of the surrounding medium,  $n$ , or its stratification or the fractional energy in the magnetic field,  $\epsilon_B$  (Kumar, 2000). The flux has a weak dependence on  $n$  when the electron cooling is dominated by the inverse Compton scattering (Panaitescu & Kumar 2000).

Under these, rather general, conditions the standard synchrotron fireball model implies that the isotropic equivalent flux at frequency  $\nu$  above the cooling frequency, at a fixed elapsed time since the explosion, is given by (Kumar 2000; Freedman & Waxman, 2001):

$$L_x = \eta_p \left[ \frac{dE}{d\Omega} \right]^{(p+2)/4} \epsilon_e^{p-1} \epsilon_B^{(p-2)/4}, \quad (2)$$

where  $dE/d\Omega$  is the energy per unit solid angle, and  $\eta_p$  is a constant<sup>||</sup>. Assuming that there is no correlation between the microscopic variables,  $\epsilon_e$ ,  $\epsilon_B$ ,  $p$  and  $dE/d\Omega$  we obtain from the above equation that  $\sigma_{dE/d\Omega} < \sigma_{L_x}$ . Using  $\sigma_{L_x} \approx 0.43 \pm 0.1$  for the 21 BeppoSAX bursts we find that  $\sigma_{dE/d\Omega} \leq 0.43 \pm 0.1$ .

From  $\sigma_{dE/d\Omega}$  we can now obtain  $\sigma_E$  that characterizes the distribution of the kinetic energy provided we

know  $\sigma_\theta$  using the trivial relation:  $\sigma_{dE/d\Omega}^2 = \sigma_E^2 + 4\sigma_\theta^2$ . Panaitescu & Kumar (PK01) and Frail et al. (F01) have estimated the jet opening angles from the observed (or lack of) breaks in the light curves of optical afterglow light curves. For 8 GRBs from the PK01 sample we have:  $\sigma_\theta \approx 0.31 \pm 0.06$  while a sample of 10 bursts from F01 yields  $\sigma_\theta \approx 0.28 \pm 0.05$ . If these values are representative for the whole GRB population we find a marginally viable solution within two  $\sigma$  errors of  $\sigma_E < 0.2$  (for the PK01 result) and  $\sigma_E < 0.27$  (for the F01 data); to get a viable solution we had to take both the values of  $\sigma_{L_x}$  one standard deviation above the mean and the value of  $\sigma_\theta$  one SD below the mean. This result suggests that there is a narrow energy distribution; the FWHM of  $E$  being less than a factor of 5. If  $E$  and  $\theta$  are correlated the above relation is modified i.e.  $\sigma_{dE/d\Omega}^2 = \sigma_E^2 + 4\sigma_\theta^2 - 4\zeta\sigma_E\sigma_\theta$ . Both the PK01 and the F01 data show that this correlation ( $\zeta$ ) if non-zero is weak, less than 0.35. With such a correlation the allowed energy distribution could be somewhat (but not significantly) broader.

A stronger constrain on  $\sigma_E$  can be obtained using  $\sigma_{\epsilon_e} = 0.3$  for 8 GRBs analyzed by PK01. It follows from Eq. 2 that a non-zero value for  $\sigma_{\epsilon_e}$  makes the distribution for  $dE/d\Omega$  and hence  $E$  even narrower, however to quantify this effect we need a larger data set.

We have argued before that  $E = E_{K,ad}$ , discussed here, is a rather good estimate to  $E_{rel}$  the total energy emitted by the “inner engine”. The constancy of  $E_{K,ad}$  is another indication for it

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<sup>||</sup>It is worth emphasizing that equation (2) is independent of the details of the jet structure i.e., the variation of Lorentz factor across the jet, since at about 1/2 a day after the explosion the jet Lorentz factor has dropped to  $\sim 10$  and we see a good fraction of the entire jet surface.

being a good measure of  $E_{rel}$ . The constancy of  $E_{K,ad}$  is also an indication that the assumptions that have lead to Eq. 2 are justified. Otherwise it would have been remarkable if starting from different levels of initial energy and having different amounts of energy losses the final kinetic energy of the afterglow would converge to a constant value. At present there is no way to tell whether significant amounts of additional energy is released in other forms, e.g. non relativistic particles or neutrinos. Similar arguments suggest further that this total energy emitted by the “central engine” does not vary significantly and that it is rather close to the energy estimated here.

The distribution of relativistic kinetic energy during the afterglow phase is narrow, with full width at half maximum less than one decade. These results suggest that the wide distribution of directly and indirectly determined  $E_{\gamma,iso}$  results from the distribution of beaming angles, from a variation of  $dE/d\Omega$  across the jet, and from a variable efficiency in conversion of kinetic energy to  $\gamma$ -rays. The fact that GRB engines are “standard” engines in terms of their energy output provide a very severe constraint on the nature of these enigmatic explosions. For instance, in the collapsar model for GRBs the central engine is composed of a black hole (BH) and an accretion disk around it (Woosley 1993; Paczynski, 1998; MacFadyen & Woosley, 1999). This model has two energy reservoirs which can be tapped to launch a relativistic jet: the BH rotation energy and the gravitational energy of the disk. Our result of nearly constant energy in GRBs implies that the mass accretion on to the BH plus the possible conversion of rotational energy of the BH to kinetic energy of the jet does not vary much from one burst to another inspite of the fact that both the disk mass and the BH spin are expected to vary widely in the collapse of massive stars.

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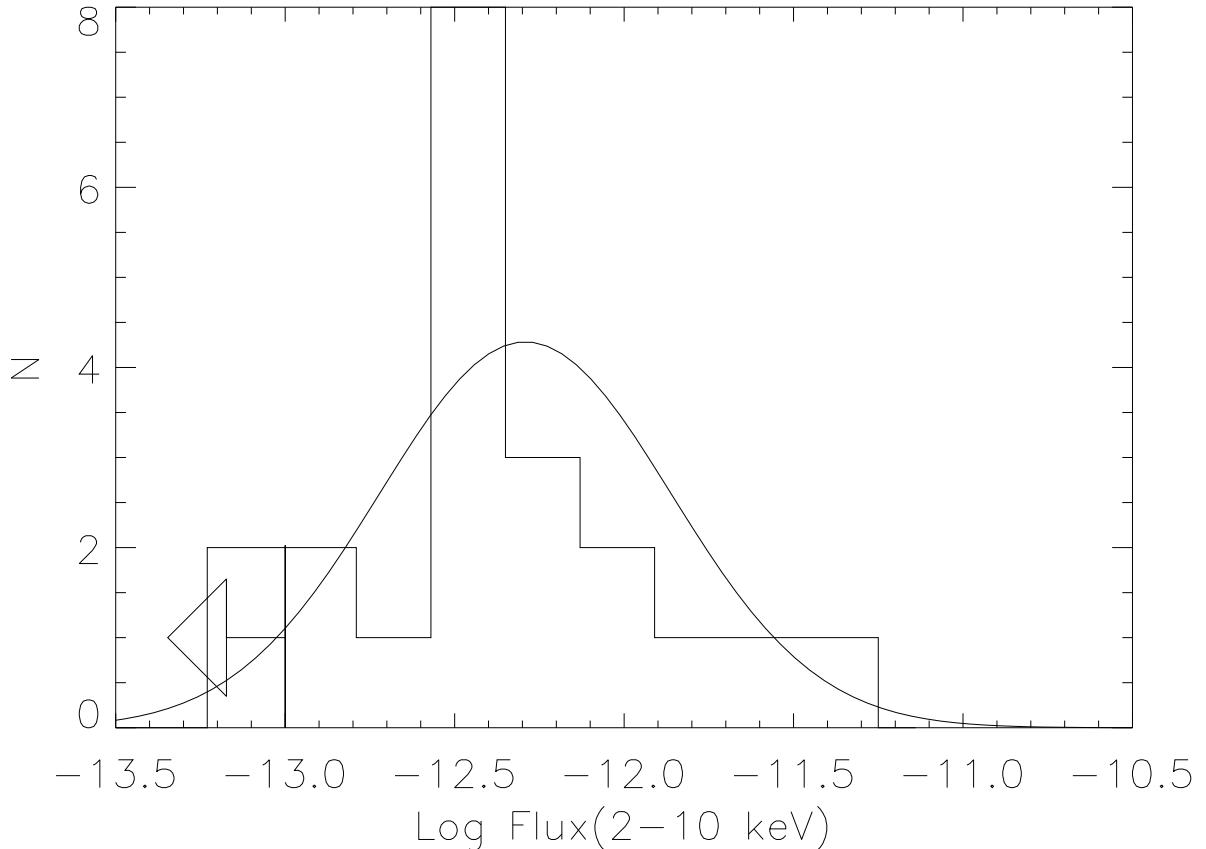


Fig. 1.— The distribution of X-ray fluxes (2-10 keV) at  $t=11$  hours after the GRB in 21 afterglows observed by BeppoSAX. The sample includes all the fast observations performed by BeppoSAX on GRB from January 1997 to October 1999. Data are from Piro (2001, proc. of X-ray astronomy 99, and references therein) Stratta et al. (2001, in preparation), Nicastro et al. 1999 (IAUC7213), Feroci et al. (2001 A&A, in press), Montanari et al. (2001, Proc. of the 2nd wokshop on GRB in the afterglow era, Rome, 17-20 Oct. 200, E. Costa, F. Frontera J. Hjort eds., in press), Piro et al. (1999, GCN 409) and T Zand et al. (2000, ApJ, 545, 266). No X-ray afterglow was detected in GB990217 (Piro et al. 1999, IAUC 7111) to the limiting instrumental sensitivity of  $10^{-13}$  erg  $\text{cm}^{-2}$   $\text{s}^{-1}$ , 6 hours after the burst. In the case of GB970111 (Feroci et al. 1998 A&A 332,L29) a candidate was detected, but evidence of fading behaviour is marginal, so we have considered both cases as upper limits (indicated by the arrow in fig).

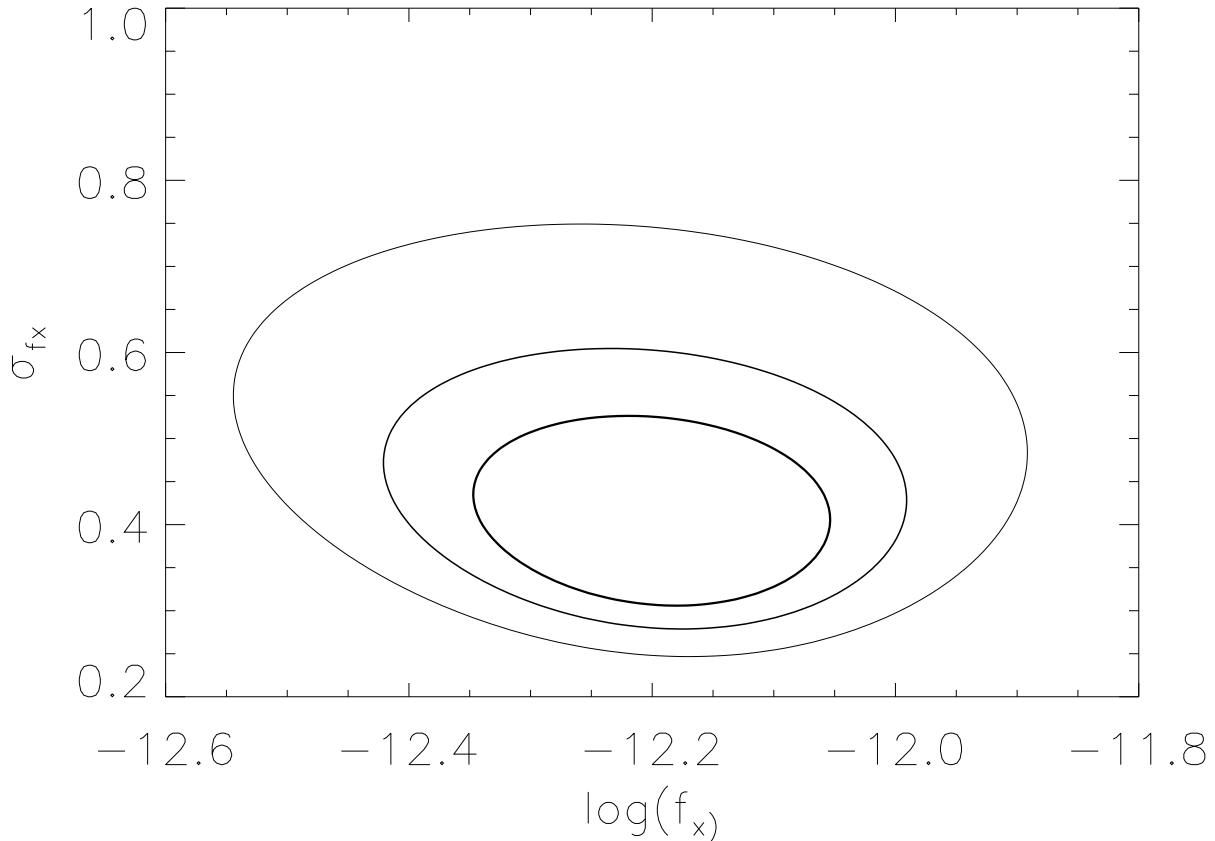


Fig. 2.— Likelihood contour lines (corresponding to 99%, 90% and 69% confidence levels) in the  $\log(f_x)$ ,  $\sigma_{f_x}$  plane for the X-ray flux distribution as inferred from 21 GRBs detected by BeppoSAX from January 1997 to October 1999. We determine  $\log(f_x)$  and  $\sigma_{f_x}$  by minimizing the likelihood function  $S = -\sum_i \ln \left\{ [2\pi(\sigma_i^2 + \sigma_{f_x}^2)]^{-1/2} \exp [-(\log f_{x_i} - \log f_x)^2 / 2(\sigma_i^2 + \sigma_{f_x}^2)] \right\}$ ; where  $\log f_{x_i}$  and  $\sigma_i$  are the observed x-ray flux 11 hr after the onset of the  $i$ -th GRB and the associated measurement error respectively. The maximal likelihood is at  $\log(f_x) = -12.2 \pm 0.2$  and  $\sigma_{f_x} = 0.43^{+.12}_{-.11}$ . Two upper limits of  $10^{-13}$  ergs  $\text{cm}^{-2} \text{ s}^{-1}$  in the 2–10 kev band at 11 hours after the bursts are included in this data set. The value of  $\sigma_{f_x}$  is 0.42 if we exclude these upper limits. We have checked a posteriori with a Kolgomorov-Smirnov test that the distribution is consistent with a gaussian (at 90% confidence level). We also note that the predicted number of X-ray afterglows with a flux lower than about  $2 \times 10^{-13}$  is 3.5, consistent with the observed number of objects.